FERTILITY ESTIMATES FROM BIRTH STATISTICS

By: K. R. Gabriel, Hebrew Univ., Jerusalem and Univ. of N. C. and Ruma Falk, Hebrew Univ., Jerusalem

I. THE NEED FOR MEASURES OF FERTILITY BASED ON BIRTH STATISTICS ONLY, AND THEIR INVESTIGATION

Refined techniques of fertility analysis have received much attention in recent years. They usually require detailed statistics both of births and of the composition of the female population. For many populations and sections for which fertility analysis is of interest -- such as occupational and ethnic groups -- such data are not usually available and simpler, if cruder, methods are still of importance.

Indices of fertility based on statistics of births only, i.e., not requiring data on the population, are of particular usefulness. This is because statistics of births (or confinements) can often be broken down by social, economic, residence, origin or other characteristics of the population for which no census data on age and sex composition are available. Such indices will be specially useful if their sampling errors can be estimated, thereby allowing their use in relatively small sub-groups of the population. A number of such indices are proposed and investigated in this study.

The validity of indices of fertility and other demographic indices is usually inferred from the logical implications of their methods of computation. Empirical validation is not often attempted, largely because statistically measurable ultimate criteria of fertility are rarely available -- except perhaps where cohort analyses have been completed. Indirect validation of indices may be obtained by correlating them with such time-honored measures of that of Total Fertility, i.e., age-of-mother-specific-birth-rates summed for all ages. Total fertility is, however, known to be prone to considerable short term fluctuations, yet neither a precise evaluation of its validity nor an acceptable method of smoothing its fluctuations is available. Critical examination of Total Fertility and similar measures based on age-specific-birth-rates is therefore indicated as a preliminary to the use of such measures as validating criteria.

Empirical investigation should ideally be carried out on a random sample from the universe of populations and dates for which validity is being studied. This is hardly practicable in demography where the sample is usually determined by the availability of statistics. In this study fertility indices have been computed and investigated on data for Australia for the 47 years 1909-1955 (this is apparently the longest available sequence of birth statistics cross classified by age of mother and order of birth). Further data are available for the Jewish population of Israel and its main origin sub-groups, though only for very few years. The Australian data give information about the behavior of the indices in time, whereas the Israeli data add some information about the characteristics of the indices in different populations of very different fertility (i.e., the origin sub-groups). The sources of the data are described in detail in Appendix II to this paper.

II. TOTAL FERTILITY AND TOTAL MATERNAL FERTILITY

Some notation will be introduced, all symbols referring to events in some given year. Denote the number of women of age a (or age group a) in the population by $P(a)$; also the number of births of order i, and of all births, by $B_i(a)$ and $B(a)$, respectively. The age-specific-birth-rate for mothers of age a is denoted by $f(a)$ and defined as

$$f(a) = \frac{B(a)}{P(a)}$$

for all births,
and as

$$f_i(a) = \frac{B_i(a)}{P(a)}$$

for births of order i.
Thus

$$\sum_i f_i(a) = f(a).$$

($\sum_i$ and $\sum_a$ are used to indicate summation over all birth orders or all age groups, respectively). Total Fertility of all births is denoted by F and defined as

$$F = \sum_a f(a).$$

(The Gross Reproduction Rate is 0.485 F, where 0.485 is the proportion of females among births.) Similarly, Total Fertility of i-th births is denoted by F_i and defined as

$$F_i = \sum_a f_i(a),$$

whence

$$F = \sum_i F_i.$$

Total Fertility -- or the Gross Reproduction Rate -- is widely accepted as a fertility measure because it is age standardized and because it would measure the average number of births per mother precisely if there were no variations in age-specific-birth-rates. By the same reasoning total fertility of i-th births -- F_i -- would measure the proportion of women having i-th births. Also since the number of mothers corresponds to the number of first births (though not necessarily every year) F/F_1 would measure the average number of births per mother and thus qualify as a measure of total maternal fertility.

True total maternal fertility is presumably very highly correlated with true total fertility, as the proportion of childless women -- which determines the difference between the two values -- probably varies very closely inversely to fertility. However, year to year fluctuations in birth rates affect both the measure of total fertility F and that of total maternal fertility F/F_1, but not always in the same way, so that the correlation between the two measures is reduced. It is difficult to judge generally whether F or F/F_1 would be more affected by fluctuations, though it might be thought that in F/F_1 fluctuations in the numerator and denominator might cancel out. As the validity and variability (due to fluctuations) of neither measure has been adequately investigated and as the logical construction of both is much the same, there seems no a priori reason to prefer either one as a measure of fertility.

The sequences of F and F/F_1 for Australia and for Israel are presented in Appendix Tables A-1 and A-2, respectively. The correlation between them is $r = 0.542$ for Australia and $r = 0.883$ for Israel with standard errors of estimate of F given F/F_1 being 0.352 for Australia and 0.515 for Israel (Tables 2 and 4). Inspection of the sequences shows that the correlation is due to the correspondence of the trends in Australia and the correspondence of the origin differentials in Israel. Short term fluctuations in the two indices differ markedly in each country, as for instance in the great depression when first births were relatively scarce and F/F_1 increased though F decreased a good deal (Australia). Undoubtedly, F/F_1 cannot be used to indicate changes in birth rates, but this does not necessarily impair its validity as a measure of fertility.

It must be remarked that it is not possible to eliminate these fluctuations by any of the standard statistical techniques. For these fluctuations are clearly neither random nor even of a simple stochastic character. Indeed for Australia it is not judged possible even to estimate the relative variability of the measures as it is not clear to what extent such variations as the fall of birth rates in the early thirties and their subsequent recovery should be considered fluctuations or real changes in fertility. For a period of relative stability in fertility -- though not in birth rates -- in Israel, i.e., for 1938-49, relative variability ($\sigma/\bar{x}$) was found to be 18°/o for F and 11°/o for F/F_1.

It is to be hoped that analysis by cohorts may solve some of these problems and provide some sort of final criterion for validation of fertility measures. Until such an analysis is available no final decision about the validity or superiority of either F or F/F_1 seems possible, hence both are used in this study.

III. RATIOS OF ALL BIRTHS TO FIRST BIRTHS

The use of birth statistics for fertility analysis is based on the distribution of births by order. Thus one expects high fertility to be associated with relatively many higher order births. Possible distortions due to the age composition of mothers may be corrected by considering each age group separately and then possibly averaging them with suitable weights. Two approaches are studied here, that using the ratio of all births to first births and that using mean birth order.

Estimation of maternal fertility by calculation of the ratio of all births to first births from statistics of confinements seems to have been first advocated by Gini (1934). This calculation can be relied on to give a correct estimate of fertility only if numbers of births and of first births do not vary much from year to year. Denoting Gini's index by S_g, we have

$$S_g = \frac{\sum_a B(a)}{\sum_a B_1(a)} = \frac{\sum_a P(a)f(a)}{\sum_a P(a)f_1(a)}$$

and this shows the index's dependence on the age composition of the female population. As age composition is irrelevant to fertility, S_g cannot be considered a satisfactory index of fertility.

Ratios of all births to first births of mothers of a specific age are obviously independent of the female population's age composition. Denote them by $S(a)$ so that

$$S(a) = \frac{B(a)}{B_1(a)} = \frac{f(a)}{f_1(a)} .$$

The ratio of all births to first births for all ages can then be expressed as a weighted mean of the age specific ratios. Either as an arithmetic mean

$$\frac{\sum_a w_1(a)\ S(a)}{\sum_a w_1(a)} ,$$

or as a harmonic mean

$$\frac{\sum_a w(a)}{\sum_a w(a)\ S^{-1}(a)} ,$$

where $w_1(a)$ and $w(a)$ are weights.

A special case of these means is Gini's index S_g, being the arithmetic mean with weights $w_1(a) = B_1(a)$ and the harmonic mean with weights $w(a) = B(a)$. Another special case is total maternal fertility

$$\frac{F}{F_1} = \frac{\sum_a f(a)}{\sum_a f_1(a)} .$$

This is given by the arithmetic mean with weights $w_1(a) = f_1(a)$, and by the harmonic mean with weights $w(a) = f(a)$.

These weighted means are of importance when the true weights are unknown and assumed weights are used instead. Of course only the proportional distribution of the weights matters, and small differences are unlikely to affect the weighted means very much. Thus if $S(a)$ can be computed from birth statistics, an estimate of maternal fertility can be obtained by averaging the $S(a)$ with appropriate weights.

Hajnal (1948) proposed using as weights the rates of another population, preferably one with similar fertility patterns to those of the population studied. (Hajnal's study is in terms of ratios and rates specific to marriage duration. Our use of ratios and rates specific to age of mother (Gabriel, 1953) is analogous, though the results may differ). Thus if a bar denotes the rates and ratios from some population used as a standard, the weights would be

$$\bar{w}_1(a) = \bar{f}_1(a)/\bar{F}_1$$

and

$$\bar{w}(a) = \bar{f}(a)/\bar{F} \ .$$

The two estimates of fertility according to Hajnal thus are: the arithmetic mean

$$S_f = \sum_a \bar{w}_1(a)\ S(a) \ ,$$

and the harmonic mean

$$S_a = \left\{\sum_a \bar{w}(a)\ S^{-1}(a)\right\}^{-1} .$$

Evidently if age-specific-birth rates in the standard population are proportional to those in the population studied, i.e.,

if $\bar{w}_1(a) = f_1(a)/F_1$

and $\bar{w}(a) = f(a)/F$,

then $S_f = S_a = F/F_1$.

Ratios of all births to first births increase with age of mother. Hence if the standard weights are concentrated at higher ages than the unknown true weights, then the means will become too large, and conversely for standard weights concentrated at lower ages.

It might be surmised that generally the less fertile a population is, the later the ages at which first births are concentrated, and the earlier the ages of all births. The latter might be explained by the fact that family limitation usually is most marked at later child-bearing ages. (Hajnal (1948) has suggested that such a relationship might hold for marriage-duration-specific-birth-rates, i.e., for marriage duration at confinement. It might of course hold for marriage duration and yet not for age, or vice versa). If these surmises are right then when the age-specific-birth-rates of some standard population are used as weights one would find

$$S_a < F/F_1 < S_f$$

if the population studied has higher fertility than the standard population, and

$$S_a > F/F_1 > S_f$$

if the population studied has lower fertility than the standard population. In either case F/F_1 would be bracketed by S_f and S_a. The actual existence of such a pattern is investigated below.

Actual computations were carried out for five year age groups of mothers. For Australian statistics two alternative sets of weights were used: (1) corresponding to high fertility years -- 1909 - 1927 average age-specific-birth-rates, and (2) corresponding to low fertility years -- 1928-1945 average rates. For Israel all computations were weighted corresponding to 1949 age-specific-birth-rates in the entire population.

Choice of weights was found to affect the actual level of S_f and S_a appreciably, but to have practically no effect at all on comparisons of S_f and S_a between times and populations. Thus for Australia the two sets of weights gave S_f values with a difference of 0.11 on the average for the 47 years and a correlation of 0.9994, and S_a values with a difference of 0.33 on the average and a correlation of 0.9992. Few corresponding data are available for Israel and for a few computations similar results were observed.

The relation between the two measures S_f and S_a is much as the relation between two differently weighted estimates of either of them. There exists a difference in level: S_f is on the average 0.12 above S_a for Australia (second set of weights) and 0.34 for Israel. The correlation between the two measures are 0.9867 for Australia and 0.9932 for Israel.

Since choice of weights affects the level of S_f and S_a and since the two measures are strongly correlated it is not surprising to find that the surmised relation between them and F/F_1 does not generally hold. In fact for both Australia and Israel the majority of observations show S_f and S_a both above or both below F/F_1. Evidently the relations between fertility and ages at confinement are not as simple as surmised above.

Though no consistent ordering of the three measures S_f, S_a and F/F_1 can be observed, the three are strongly correlated. Correlations, regressions and standard errors of estimate of F/F_1 given each of S_f and S_a, are shown in Tables 1 and 3. It appears that each of the indices can give a close estimate of F/F_1, and S_f is the better one -- the standard error of estimate of F/F_1 with respect to S_f being 0.10-0.13 for Australia and 0.22 for Israel. The corresponding correlations are about 0.98 in either country. The regression equations are rather different for the two countries, possibly due to the different weights employed.

Correlations with F are considerably lower -- Tables 2 and 4 -- and the standard error of estimate of F is about 0.35 for Australia and 0.50 for Israel. As the measures S_f and S_a were shown to correspond in their construction to F/F_1 rather than to F it is not surprising that they should be more highly correlated with the former. Also, the correlations of S_f and S_a with F are very nearly the same as those of F/F_1 with F, both for Australian and for Israeli data. One may therefore say that possible shortcomings of S_f and S_a as measures of fertility may lie in their being estimates of F/F_1 rather than of F. It is remarkable that the S_f and S_a measures seem to be practically as good for estimating F as the F/F_1 ratio is, despite the fact that the former are calculated without statistics of the age composition of the population.

IV. MEAN BIRTH ORDER

An alternative approach to the measurement of fertility from birth statistics is based on mean birth order. Though mean birth order is not a function of maternal fertility (i.e., mean number of births per mother) only, it can reasonably be assumed to be closely correlated with it.[1]

Mean birth order, denoted by μ_B, will be defined for a population with uniform age distribution in the reproductive ages (just as Total Fertility is), i.e.,

$$\mu_B = \frac{1}{F} \sum_i i \, F_i \; .$$

Introducing the expression for F_i in terms of births and population this becomes

$$\mu_B = \frac{1}{F} \sum_a \frac{1}{P(a)} \sum_i i \, B(a) \; .$$

Defining mean birth order for mothers of age a as

$$\mu_B(a) = \sum_i i \, B_i(a)/B(a)$$

we obtain

$$\mu_B = \frac{1}{F} \sum_a f(a) \; \mu_B(a) \; .$$

Now when the age-specific-birth rates f(a) are not known for a population one might substitute those of a standard population with similar fertility, $\bar{f}(a)$, say. Defining the substitute weights as $\bar{w}(a) = \bar{f}(a)/\bar{F}$ one would then obtain the estimate

$$\hat{\mu}_B = \sum_a \bar{w}\,(a) \; \mu_B(a) \; .$$

The weighting is the same as for S_a, and the reasoning that with suitable weights such an estimate be good also is similar to that for S_f and S_a. In this study $\hat{\mu}_B$ was computed only by using five-year age groups.

A possible advantage in the use of $\hat{\mu}_B$ over S_f, S_a could be in its using the information of the entire birth order distribution whereas S_f, S_a use only the proportion of first births. This might reduce the sampling error of $\hat{\mu}_B$ relative to S_f and S_a. An obvious disadvantage of $\hat{\mu}_B$ is, however, that even μ_B which it is supposed to estimate is not strictly the same as fertility. Hence $\hat{\mu}_B$ can at best only serve as an index of fertility.

Finally, some statisticians have used $\mu_B(a_s)$ of some particular age group a_s directly as an index of fertility. (We are indebted for this idea to Prof. R. Bachi of the Hebrew University, Jerusalem.) This seems a justifiable procedure as it is reasonable to suppose that variations in μ_B will generally be reflected in similar variations in $\mu_B(a)$ at all ages -- though exceptions are of course possible. This method has the attraction of easy computation and no need for assumptions about the suitability of weights but it is of course subject to possible distortion if the $\mu_B(a)$'s should behave differently at different ages. The age group chosen is usually about the middle of the reproductive ages, and in this study the use of each of the three age groups 25-29, 30-34 and 35-39 -- denoted by a_1, a_2 and a_3, respectively -- has been investigated.

For Australia $\mu_B(a_s)$ has been computed for the three age groups mentioned above, and the mean $\hat{\mu}_B$ has been calculated with weights proportional to the mean of the 1909-1955 age-of-mother-specific-birth-rates. For Israel only $\mu_B(30\text{-}34)$ is available. (See Appendix Tables A-1 and A-2.)

Correlations of the μ_B indices with F/F_1 and F -- Tables 1 to 4 -- are found to be just slightly lower than those of S_f and S_a. It is not possible to draw reliable conclusions as to the relative standing of the various indices as the observations are too few. Yet, it would seem that $\mu_B(25\text{-}29)$ is practically as good an index of maternal fertility as S_f and S_a. Also its correlation with F is much below its correlation with F/F_1, clearly for the same reasons as were discussed with regard to S_f and S_a.

The finding that $\mu_B(a)$ predicts F/F_1 best for the lowest age group, even better than $\hat{\mu}_B$ does, indicates that in the higher mother age groups mean birth order is less closely related to mean fertility. Perhaps this is because only the more fertile women bear at those ages at all.

From the purely computational point of view the $\mu_B(a)$ measures are definitely preferable to any of the weighted means. Both for the estimate itself and for its standard error (see Appendix I) the computations are greatly reduced. Also if only one age group is used, this may mean considerable saving in sorting and tabulating whenever these are done specially for the purpose of fertility analysis.

V. CONCLUSIONS

It has been shown that indices of fertility based on birth statistics alone are very highly correlated with a measure of maternal fertility. For most purposes it would seem that some of the indices are practically as reliable as the ratio F/F_1 computed from age-specific-birth-rates. The best of the indices seem to be S_f and $\mu_B(25\text{-}29)$, the latter having the additional advantage of requiring only few and simple computations, both for the estimate and for its standard error.

Indices of fertility which are computed from birth statistics are based on the distribution of births by order. They are therefore related directly to maternal fertility rather than to all fertility. This explains why their correlations with Total Fertility are much the same as those of Total Maternal Fertility.

For the lack of any available ultimate statistical criterion of fertility it cannot be said whether F -- which estimates mean number of births per woman -- or F/F_1 -- which estimates mean number of births per mother -- is the more valid measure of fertility. Hence no final evaluation of the measures based on birth statistics only is possible at this stage.

APPENDIX I

SAMPLING ERRORS

The magnitude of sampling variability is presented here by the standard error or its square, the variance. The sampling distributions have not been studied but for sizeable samples they may safely be assumed to be approximately normal.

Mean birth order estimates are clearly unbiased but the bias of the S_f and S_a estimates has not been investigated. For large samples it is likely to be negligible.

Variances were first computed for the individual terms of the various indices, i.e., for proportions of first births to all births, or mean birth order, at a given age of mother. Then they were transformed and summed to give the variances of the fertility indices.

For this derivation we used the two well known theorems that if X_i are independent variables and a_i constant weights

$$\text{Var}\ (\sum_i a_i\ X_i) = \sum_i a_i^2\ \text{Var}\ (X_i)$$

and

$$\text{Var}\left(\frac{X_1}{X_2}\right) = E\left(\frac{X_1}{X_2}\right)^2 \left\{\frac{\text{Var}\ (X_1)}{(E\ X_1)^2} + \frac{\text{Var}(X_2)}{(E\ X_2)^2}\right\} \text{approximately}$$

Further we have used the known expressions for the variance of a proportion and of a mean. However, in order to simplify the presentation we have introduced in the denominator the number of observations instead of the degrees of freedom which would have been one less.

Thus as

$$S(a) = B(a)/B_1(a)$$

$$\text{Var}(S^{-1}(a)) = \frac{(S^{-1}(a))\ (1-S^{-1}(a))}{B(a)}$$

and therefore

$$\text{Var}(S(a)) = \frac{S(a)\ (S(a) - 1)}{B_1(a)}\ .$$

Hence it is found that

$$\text{Var}(S_f) = \sum_a (\bar{w}_1(a))^2\ \frac{S(a)\ (S(a)-1)}{B_1(a)}$$

and

$$\text{Var}(S_a) = \frac{1}{S_a^4} \sum_a (\bar{w}(a))^2\ \frac{S^{-1}(a)(1-S^{-1}(a))}{B(a)}\ .$$

Also as $\quad \mu_B(a) = \sum_i i\ B_i(a)/B(a)$

we obtain $\quad \text{Var}(\mu_B(a)) = \sigma_B^2(a)/B(a)$

where $\quad \sigma_B^2(a) = \sum_i i^2\ B_i(a)/B(a) - (\mu_B(a))^2.$

For $\hat{\mu}_B$ it follows that

$$\text{Var}(\hat{\mu}_B) = \sum_a (\bar{w}(a))^2\ \frac{\sigma_B^2(a)}{B(a)}\ .$$

It should be quite clear that the above formulae refer to estimates of the standard errors which can be calculated from the sample and not to the population values. All the expressions given here can easily be calculated along with the estimate itself.

APPENDIX II

DATA AND ESTIMATES FROM WHICH FERTILITY ESTIMATES WERE COMPUTED

The measures of fertility investigated in this paper are illustrated by computations from Australian data for each of the years 1909 to 1955. The basic figures were taken from publications of the Australian Commonwealth Bureau of Statistics and supplemented by figures made available to the authors by the courtesy of that Bureau. Where figures were incomplete or did not correspond precisely to the requirements of this study, estimates and adjustments were introduced, as described in this Appendix. These estimates and adjustments were often crude but were deemed adequate for the purpose of calculating and comparing fertility measures in this paper. No other use of these estimates should be made without investigating whether they are reliable enough for such purposes.

Figures of the female population by five year age groups were available for all years since 1921. They are based on the censuses of 1921, 1933, 1947 and 1954, and on intercensal estimates. These figures were used here as published.

For the years 1909-1920 an interpolation-extrapolation procedure was used to estimate the female population. For each five year age group linear interpolation between 1911 and 1921 census figures gave a first estimate (for 1909 and 1910 the same trend was extrapolated). The interpolated figures were then adjusted proportionally so that their sum for ages 10-54 be equal to the estimated number of women at those ages in the population of that year. Since no such numbers had been published, they were estimated by multiplying the total number of females in each year (published) by the average proportion in age group 10-54 among females in the censuses of 1911 and 1921.

The basic data on births were tables of nuptial confinements by age of mother and previous issue. These figures relate only to live births and to previous issue from the present marriage. For the estimation of fertility it would have been desirable to have figures for all previous issue -- however, no adjustment has been made for this. Furthermore, ex-nuptial confinements were not included in the calculations, except that 15.2 percent of them were added to nuptial first births. (This is the percentage of legitimations to ex-nuptial births in the preceding year, computed from 1924-1955 figures.) These additional 15.2 percent of ex-nuptial live births were distributed among mothers' age groups in proportion to other first births. A small number of births for which age of mother and previous issue were unknown were also added in proportionally to other births.

These adjusted figures for births were used in the computation of birth rates. The other measures of fertility were computed directly from the unadjusted data. Omission of the legitimations (about 1 percent of the births) must thus have slightly increased both mean birth order and ratios of all births to first births. However, since this omission occurred in all years its effect must have been systematic and comparisons of trends and changes should not have been affected.

The data for Israel are taken from the first author's research on fertility in Israel (Gabriel, 1957). The calculations are based on official government statistics of births and of the population, supplemented by some estimates where data were incomplete. For details the reader is referred to Chapter 6 of the above work.

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FOOTNOTES

1. It is easily shown that, if μ_M and σ_M^2 are the mean and variance of the distribution of mothers by number of births and μ_B is mean birth order, then $\mu_B = 1/2\left\{1 + \mu_M \left[1 + \left(\frac{\sigma_M}{\mu_M}\right)^2\right]\right\}$.

Thus, mean birth order is affected by the relative variation of fertility as well as by mean fertility. Yet it would appear that relative variation of fertility does not vary as much as mean fertility and thus the correlation between μ_B and μ_M would be high.

Table 1 Correlations, Regressions and Standard Errors of Estimate of Various Measures with Total Maternal Fertility - F/F_1. (Australia 1909-55; 47 observations, F/F_1-mean 3.345, Standard Deviation .5581).

	Mean	Regression Equation	Standard Error of Estimate	Correlation
S_f(1909-27 weights)	3.509	1.0458 S_f - 0.3248	.1043	.9824
S_a(1909-27 weights)	3.609	1.2445 S_a - 1.1464	.1892	.9408
S_f(1928-45 weights)	3.396	1.1011 S_f - 0.3943	.1296	.9726
S_a(1928-45 weights)	3.281	1.4350 S_a - 1.3633	.2116	.9253
μ_B(25-29)	2.381	2.9642 μ_B - 3.7127	.0997	.9839
μ_B(30-34)	3.301	1.4711 μ_B - 1.5112	.1798	.9467
μ_B(35-39)	4.447	0.8222 μ_B - 0.3112	.2639	.8811
$\hat{\mu}_B$(1909-55 weights)	2.926	1.7767 $\hat{\mu}_B$ - 1.8536	.2109	.9258

Table 2 Correlations, Regressions and Standard Errors of Estimate of Various Measures with Total Fertility - F. (Australia 1909-1955; 47 observations, F-mean 2.659, Standard Deviation .4189)

	Mean	Regression Equation	Standard Error of Estimate	Correlation
F/F_1	3.345	.4066F/F_1 + 1.2988	.3521	.5417
S_f(1909-27 weights)	3.509	.4463 S_f + 1.0931	.3475	.5584
S_a(1909-27 weights)	3.609	.5545 S_a + 0.6577	.3475	.5584
S_f(1928-45 weights)	3.396	.4799 S_f + 1.0292	.3457	.5648
S_a(1928-45 weights)	3.281	.6488 S_a + 0.5302	.3478	.5573
μ_B(25-29)	2.381	1.2039 μ_B - 0.2075	.3546	.5323
μ_B(30-34)	3.301	0.5189 μ_B + 0.9463	.3752	.4448
μ_B(35-39)	4.447	0.2175 μ_B + 1.6917	.3982	.3105
$\hat{\mu}_B$(1909-55 weights)	2.926	0.5315 $\hat{\mu}_B$ + 1.1039	.3894	.3689

TABLE A-1. VARIOUS INDICES OF FERTILITY - AUSTRALIA 1909-1955.

	F	F/F_1	$S_f^{(1)}$	$S_f^{(2)}$	$S_a^{(1)}$	$S_a^{(2)}$	$\mu_B(a_1)$	$\mu_B(a_2)$	$\mu_B(a_3)$	$\hat{\mu}_B$
1909	3.178	4.406	4.425	4.238	4.216	3.780	2.665	3.839	5.412	3.382
1910	3.211	4.407	4.460	4.271	4.245	3.805	2.667	3.873	5.316	3.375
1911	3.249	4.149	4.173	4.008	4.069	3.658	2.634	3.803	5.266	3.330
1912	3.405	4.015	4.109	3.950	4.023	3.620	2.629	3.797	5.232	3.318
1913	3.339	3.864	3.974	3.824	3.939	3.552	2.602	3.738	5.144	3.270
1914	3.301	3.891	3.956	3.815	3.982	3.597	2.599	3.730	5.151	3.271
1915	3.155	3.886	3.963	3.819	3.984	3.599	2.582	3.703	5.110	3.249
1916	3.028	3.911	4.021	3.873	4.006	3.615	2.565	3.675	4.995	3.217
1917	2.950	4.291	4.304	4.149	4.286	3.862	2.625	3.732	5.084	3.271
1918	2.800	4.501	4.526	4.358	4.445	3.993	2.659	3.747	5.010	3.281
1919	2.670	4.282	4.354	4.194	4.222	3.786	2.655	3.746	5.016	3.262
1920	2.911	3.443	3.560	3.424	3.535	3.193	2.476	3.571	4.940	3.122
1921	2.893	3.316	3.387	3.263	3.429	3.107	2.429	3.553	4.935	3.090
1922	2.900	3.586	3.678	3.546	3.727	3.374	2.449	3.555	4.931	3.115
1923	2.815	3.620	3.677	3.552	3.779	3.428	2.451	3.504	4.844	3.091
1924	2.773	3.648	3.731	3.606	3.832	3.474	2.454	3.492	4.829	3.090
1925	2.739	3.637	3.782	3.647	3.838	3.474	2.475	3.513	4.831	3.103
1926	2.638	3.566	3.700	3.578	3.797	3.441	2.483	3.479	4.751	3.079
1927	2.594	3.480	3.673	3.541	3.727	3.374	2.468	3.451	4.741	3.066
1928	2.554	3.406	3.594	3.473	3.684	3.341	2.448	3.474	4.685	3.052
1929	2.437	3.417	3.608	3.490	3.717	3.375	2.448	3.436	4.650	3.038
1930	2.389	3.407	3.592	3.474	3.717	3.377	2.445	3.391	4.638	3.016
1931	2.179	3.464	3.682	3.562	3.800	3.450	2.460	3.443	4.715	3.051
1932	2.019	3.549	3.811	3.683	3.898	3.531	2.475	3.442	4.683	3.061
1933	2.002	3.414	3.689	3.556	3.751	3.399	2.421	3.410	4.598	3.018
1934	1.947	3.233	3.476	3.350	3.552	3.226	2.381	3.344	4.604	2.981
1935	1.958	3.037	3.257	3.138	3.359	3.059	2.332	3.241	4.489	2.917
1936	2.021	2.874	3.026	2.922	3.173	2.901	2.249	3.114	4.354	2.826
1937	2.046	2.826	3.079	2.957	3.158	2.886	2.218	3.083	4.339	2.805
1938	2.048	2.765	2.876	2.786	3.077	2.827	2.180	2.992	4.235	2.747
1939	2.065	2.708	2.798	2.717	3.028	2.789	2.154	2.948	4.096	2.701
1940	2.150	2.639	2.747	2.669	2.980	2.747	2.135	2.871	4.020	2.652
1941	2.213	2.588	2.677	2.603	2.909	2.682	2.101	2.846	3.896	2.603
1942	2.222	2.593	2.703	2.628	2.922	2.687	2.105	2.844	3.838	2.584
1943	2.400	2.539	2.625	2.558	2.853	2.627	2.085	2.797	3.732	2.531
1944	2.459	2.825	2.876	2.805	3.099	2.844	2.143	2.825	3.671	2.540
1945	2.565	2.822	2.883	2.811	3.098	2.841	2.142	2.826	3.619	2.527
1946	2.804	2.725	2.811	2.743	3.039	2.792	2.119	2.798	3.587	2.491
1947	2.871	2.585	2.763	2.696	2.973	2.728	2.103	2.793	3.592	2.477
1948	2.772	2.816	3.036	2.971	3.269	3.003	2.188	2.853	3.594	2.528
1949	2.769	2.897	3.184	3.117	3.410	3.130	2.200	2.915	3.619	2.556
1950	2.846	2.985	3.288	3.219	3.525	3.237	2.229	2.918	3.631	2.573
1951	2.832	3.005	3.365	3.292	3.598	3.299	2.256	2.930	3.669	2.598
1952	2.945	2.963	3.354	3.280	3.581	3.281	2.282	2.973	3.676	2.617
1953	2.955	3.008	3.442	3.368	3.667	3.360	2.314	3.011	3.707	2.648
1954	2.953	3.084	3.558	3.482	3.789	3.469	2.345	3.036	3.764	2.676
1955	3.005	3.147	3.692	3.612	3.912	3.577	2.394	3.077	3.777	2.709

Notes $S_f^{(1)}$, $S_a^{(1)}$ computed with 1909-27 average birth rates as weights, $S_f^{(2)}$, $S_a^{(2)}$ with 1928-45 average rates. a_1, a_2 and a_3 refer to age groups 25-29, 30-34 and 35-39, respectively.

Table A-2 VARIOUS INDICES OF FERTILITY - ISRAEL: ALL WOMEN 1938-1954, WOMEN BY CONTINENT OF BIRTH 1938/40, 1944/45, 1949, 1951-1954.

	F	F/F_1	S_f	S_a	$\mu_B(30\text{-}34)$
All Women					
1938	2.48	2.23	2.24	2.24	
1939	2.23	2.20	2.21	2.22	
1940	2.35	2.17	2.15	2.17	
1941	2.12	2.05	1.98	2.03	
1942	2.38	2.00	1.93	2.01	
1943	3.11	2.13	1.98	2.07	
1944	3.44	2.64	2.38	2.41	2.50
1945	3.53	2.81	2.51	2.56	2.58
1946	3.34	2.72	2.52	2.53	2.63
1947	3.54	2.46	2.37	2.35	2.76
1948	3.08	2.32	2.25	2.27	2.73
1949	3.43	2.45	2.44	2.45	2.76
1950	3.90	3.05	3.09	2.99	3.12
1951	4.01	3.04	3.45	3.25	3.44
1952	3.98	3.29	3.68	3.51	3.61
1953	3.88	3.37	3.88	3.69	3.69
1954	3.59	3.63	4.20	3.97	3.75
Israel-born Women					
1938/40	3.54	3.85	4.15	3.59	4.60
1944/45	3.87	3.95	3.56	3.13	3.69
1949	3.56	3.18	3.22	2.63	3.86
1951	3.56	3.46	3.65	3.12	3.80
1952	3.36	3.50	3.75	3.15	3.61
1953	3.22	3.39	3.61	3.06	3.70
1954	2.89	3.44	3.68	3.10	3.54
Women born in Asia and Africa					
1938/40	4.62	5.25	6.84	5.86	5.61
1944/45	4.99	4.71	5.46	4.50	4.84
1949	4.45	4.01	5.42	4.29	4.72
1951	6.30	4.53	6.12	4.86	5.07
1952	6.23	5.07	6.95	5.45	5.21
1953	6.15	5.13	6.85	5.66	5.27
1954	5.67	5.56	7.62	6.21	5.28
Women born in Europe, etc.					
1938/40	1.85	1.73	1.67	1.72	2.10
1944/45	3.06	2.32	2.00	2.04	2.04
1949	3.21	2.11	2.06	2.10	2.22
1951	3.17	2.33	2.57	2.51	2.25
1952	3.04	2.38	2.58	2.55	2.28
1953	2.87	2.39	2.70	2.61	2.31
1954	2.63	2.48	2.81	2.74	2.39

Table 3 Correlations, Regressions and Standard Errors of Estimate of Various Measures with Total Maternal Fertility - F/F_1. (Israel, total population and origin groups 1938-1954; 38 observations, F/F_1 Mean 3.140, Standard Deviation 1.030).

	Mean	Regression Equation	Standard Error of Estimate	Correlation
S_f	3.488	$.6213\ S_f + .9733$	.2247	.9759
S_a	3.153	$.8511\ S_a + .4571$	.2537	.9692
$\mu_B(30\text{-}34)$*	3.499	$.8985\ \mu + .1862$	.2644	.9655

(* only 32 observations, F/F_1 mean 3.330, F/F_1 St. Dev. 1.016)

Table 4 Correlations, Regressions and Standard Errors of Estimate of Various Measures with Total Fertility - F. (Israel, total population and origin groups 1938-1954; 38 observations, F-mean 3.596, Standard Deviation 1.096).

	Mean	Regression Equation	Standard Error of Estimate	Correlation
F/F_1	3.140	$.9390 F/F_1 + .6468$	.5148	.8828
S_f	3.488	$.6053\ S_f + 1.4843$	.4912	.8940
S_a	3.153	$.8301\ S_a + .9786$	.5025	.8837
$\mu_B(30\text{-}34)$*	3.499	$.8044\ \mu + .9970$	.5825	.8333

(* only 32 observations, F-mean 3.811, Standard Deviation 1.054)